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Thermally stimulated current transport peaks in insulating layers with spatially non-homogeneous trap distribution: II. Bulk initial carrier generation

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Abstract. In the present paper we continue the recent investigations of Tomaszewicz *et al* on the influence of spatial non-homogeneity of the trap distribution on thermally stimulated currents (TSC) due to transport of carriers in insulating layers. We consider here—in contrast to the previous work—the case of *bulk* initial generation of carriers. For both non-dispersive and dispersive transport, the analytical formulae obtained agree satisfactorily with the results of Monte Carlo simulation. The possibility of determining the spatial trap distribution on the basis of the measured TSC is discussed.

1. Introduction

In the previous work (Tomaszewicz *et al* 1990) we discussed the influence of the spatial non-homogeneity of the trap distribution on the thermally stimulated current (TSC) transport peaks in the case of surface generation of carriers at the initial moment of time (thermally stimulated time of flight (TOF)). The formulae obtained therein for TSC generalize the earlier results for a spatially homogeneous trap distribution over the layer thickness (e.g. Plans *et al* 1981, Tomaszewicz and Jachym 1984). For both non-dispersive and dispersive transport the initial increase of TSC has been found to be strongly dependent on the spatial distribution of traps. The analytical results allow the determination of the spatial trap distribution on the basis of the measured TSC.

In many TSC experiments, however, the carrier density is distributed over the whole sample volume at the onset of the measurement. In the present paper we consider two specific cases: (i) initial trapped carrier density proportional to the trap density at a given point; (ii) uniform initial distribution of trapped carriers. The initial condition (i) can be realized by carrier photogeneration near the sample surface at very high fields, when the carrier mean free path $\mu_0\tau_t E$ (μ_0 = free carrier microscopic band mobility, τ_t = free carrier lifetime, E = external electric field) significantly exceeds the sample thickness.

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As will be shown, the formulae describing TSC then take a relatively simple form. The initial condition (ii) corresponds to volume carrier generation by weakly absorbed light. This case has already been studied by the Monte Carlo method (Rybicki *et al* 1989).

The present paper can be considered as a continuation of the previous one (Tomaszewicz *et al* 1990), to which we refer the reader for the detailed derivation of the transport equations, as well as for a more extensive literature review.

2. Analytical considerations

After a general formulation of the problem under consideration (section 2.1), we deal in detail with specific cases of non-dispersive (section 2.2) and dispersive (section 2.3) transport.

2.1. General formulation

Let us consider the analytical description of the TSC transport peaks in non-homogeneous insulating layers. We write the trap density per unit energy in the following form:

$$N_t(x, \mathscr{E}) = S(x)N(\mathscr{E}) \tag{1}$$

where the dimensionless function $S(x)$ determines the spatial trap distribution ($x =$ space coordinate), while the function $N(\mathscr{E})$ describes the energetic trap distribution; $\mathscr{E} =$ trap depth measured downward from the edge of the conduction band (electrons) or upward from the valence band (holes). Although a more general trap distribution could also be dealt with (e.g. $N_t(x, \mathscr{E}(x))$; cf Rybicki and Chybicki 1990), we confine ourselves to the factoralized form (1), believing it to represent a sufficient—from a practical point of view—range of possible trap distributions. The free and trapped carrier concentrations will be denoted by $n(x,t)$ and $n_t(x,t)$, respectively. The inequalities $n(x,t) \ll n_t(x,t)$ and $\partial n(x,t)/\partial t \ll \partial n_t(x,t)/\partial t$ are assumed to be fulfilled. The continuity equation takes the simplified form:

$$\partial n(x,t)/\partial t + \mu_0 E\, \partial n(x,t)/\partial x \simeq 0. \tag{2}$$

The approximate equation determining the trapping/detrapping kinetics depends on the carrier transport regime, and will be specified later (sections 2.2 and 2.3 for non-dispersive and dispersive transport, respectively).

As mentioned in the introduction, the initial conditions for the transport equations are chosen to be

$$n_t(x, 0) = n_0 S(x)/S_{av} \tag{3a}$$

(case (i)), or

$$n_t(x, 0) = n_0 \tag{3b}$$

(case (ii)), where n_0 is the initial carrier density averaged over the sample thickness L, and S_{av} is the average value of $S(x)$:

$$S_{av} = \frac{1}{L}\int_0^L S(x)\,\mathrm{d}x. \tag{4}$$

We assume no carrier injection into the solid for $t > 0$, which corresponds to the boundary condition

$$n(0, t) = 0 \qquad t > 0. \tag{5}$$

The TSC induced by the carrier motion in the sample is given by

$$I(t) = \frac{I_0}{n_0 L} \int_0^L n(x, t)\, \mathrm{d}x \tag{6}$$

where $I_0 = en_0\mu_0 EA$ (e = elementary charge, A = contact surface).

2.2. Non-dispersive transport

In the case of non-dispersive transport the free and trapped carrier densities are interrelated by the approximate formula (Tomaszewicz *et al* 1990):

$$n_t(x, t) \simeq S(x)n(x, t)/\theta(t) \tag{7}$$

where the function $\theta(t)$ is given by

$$\theta^{-1}(t) = C_t \int_{\mathscr{E}_t^0}^{\mathscr{E}_t} N(\mathscr{E})\tau_d(t, \mathscr{E})\, \mathrm{d}\mathscr{E}. \tag{8}$$

Here C_t is the carrier capture coefficient, $\mathscr{E}_t^0$ and $\mathscr{E}_t$ are the limits of the trap distribution, and $\tau_d(t, \mathscr{E})$ is the carrier mean dwell time in a trap of depth $\mathscr{E}$:

$$\tau_d(t, \mathscr{E}) = \nu_0^{-1} \exp[\mathscr{E}/kT(t)] \tag{9}$$

where ν_0 is the frequency factor, k is the Boltzmann constant and $T(t)$ is the sample temperature at the moment t. The general solutions of the transport equations (2) and (7) with the boundary condition (5) are

$$n(x, t) = \theta(t)f[z(x) - \zeta(t)]H[z(x) - \zeta(t)] \tag{10}$$

$$n_t(x, t) = S(x)f[z(x) - \zeta(t)]H[z(x) - \zeta(t)] \tag{11}$$

where f is an arbitrary function and H is the unit step function. The functions $z(x)$ and $\zeta(t)$ are defined by

$$z(x) = \frac{1}{\mu_0 E} \int_0^x S(x')\, \mathrm{d}x' \tag{12}$$

$$\zeta(t) = \int_0^t \theta(t')\, \mathrm{d}t'. \tag{13}$$

The shape of the function f depends on the initial condition as well as on the spatial trap distribution. Particularizing equation (11) to $t = 0$ we get an implicit equation for function f:

$$S(x)f[z(x)] = n_t(x, 0). \tag{14}$$

In general, to solve this equation the shape function $S(x)$ must be specified. However, if the initial density of trapped carriers is proportional to the trap density, according to (3*a*) (case (i)), the above equations imply immediately $f(z) = n_0/S_{av}$. Therefore

$$n(x,t) = n_0[\theta(t)/S_{av}]H[x - \bar{x}(t)] \tag{15}$$

$$n_t(x,t) = n_0[S(x)/S_{av}]H[x - \bar{x}(t)] \tag{16}$$

where the function $\bar{x}(t)$ is given implicitly by

$$z[\bar{x}(t)] = \zeta(t). \tag{17}$$

The function $\bar{x}$ describes the position of the left boundary of the carrier packet. As follows from equation (6), the resulting TSC equals

$$I(t) = I_0[\theta(t)/S_{av}][1 - \bar{x}(t)/L]H[L - \bar{x}(t)]. \tag{18}$$

The TSC (18) is easily seen to depend on the spatial trap distribution only via the function $\bar{x}(t)$. For the initial stage of the carrier transport, $\bar{x}(t) \ll L$, the current increases, and $I(t)$, being proportional to $\theta(t)$, is independent of the spatial distribution of trapping sites. The moment τ_e, at which the last carrier leaves the sample and, correspondingly, the current drops to zero, is determined by the formula $\bar{x}(\tau_e) = L$, or equivalently by

$$\zeta(\tau_e) = \tau_0 S_{av} \tag{19}$$

where $\tau_0 = L/\mu_0 E$ is the trap-free time of flight. Formula (19) has an identical form to the case of surface carrier generation (Tomaszewicz *et al* 1990), and thus the form of the dependence of τ_e on the ratio L/E and on the heating rate β is essentially independent of the spatial trap distribution.

In order to proceed with case (ii), for the initial trap density (3*b*), an explicit form of $S(x)$ must be known. The formulae corresponding to a specific case of an exponential dependence of $S(x)$ are given in appendix 2.

2.3. *Dispersive transport*

Let us now consider the case of dispersive transport. The approximate relation between free and trapped carrier density assumes the form (Tomaszewicz *et al* 1990):

$$\frac{\partial}{\partial t}\left(\frac{n_t(x,t)}{\Phi(t)}\right) = S(x)n(x,t) \tag{20}$$

$$\Phi(t) = C_t \int_{\mathscr{E}_0(t)}^{\mathscr{E}_t} N(\mathscr{E})\,\mathrm{d}\mathscr{E}. \tag{21}$$

The demarcation level $\mathscr{E}_0(t)$ is determined by

$$\int_0^t \frac{\mathrm{d}t'}{\tau_d[t', \mathscr{E}_0(t)]} = 1. \tag{22}$$

The general solutions of equations (2) and (20), corresponding to the boundary condition (5), are

$$n(x,t) = \partial g[z(x), t]/\partial t \tag{23}$$

$$n_t(x,t) = \Phi(t)S(x)g[z(x), t] \tag{24}$$

where

$$g(z,t)=\int_0^z h(z-z')\exp[-z'\Phi(t)]\,\mathrm{d}z'. \tag{25}$$

The function $z(x)$ is given by equation (12), and the form of the function h is determined by initial conditions. In order to simplify further calculations we assume that $z(L)\Phi(0) \gg 1$. Then, at the initial moment $t=0$, the value of the function h at the point $z-z'$ in (25) can be replaced by its value at the point z, which yields

$$g(z,0) \simeq h(z)/\Phi(0). \tag{26}$$

Setting $t=0$ in equation (24) and making use of equation (26) we get the formula determining the function $h(z)$:

$$S(x)h[z(x)] \simeq n_t(x,0). \tag{27}$$

One can note that under the approximation considered the function $h(z)$ becomes identical to the function $f(z)$, introduced for the description of non-dispersive transport (cf equations (14) and (27)). In consequence, a relatively simple solution of the transport equations can be obtained only for initial condition (3a), when $n_t(x,0) \sim S(x)$ (case (i)). In this case we have $h(z) \simeq n_0/S_{av}$, and thus from equations (23)–(25) one obtains

$$n(x,t)=\frac{n_0}{S_{av}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\Phi(t)}\{1-\exp[-z(x)\Phi(t)]\}\right) \tag{28}$$

$$n_t(x,t)=n_0[S(x)/S_{av}]\{1-\exp[-z(x)\Phi(t)]\}. \tag{29}$$

According to (6) the TSC is

$$I(t)=\frac{I_0}{LS_{av}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\Phi(t)}\int_0^L\{1-\exp[-z(x)\Phi(t)]\}\,\mathrm{d}x\right). \tag{30}$$

Let us discuss some limiting cases of the above formula. For the initial stage of carrier transport, when $z(L)\Phi(t)=\tau_0 S_{av}\Phi(t) \gg 1$, the second term in the integrand can be ignored, which results in

$$I(t) \simeq \frac{I_0}{S_{av}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\Phi(t)}\right). \tag{31}$$

On the other hand the exponential function in the integrand can be resolved into a power series only if $z(L)\Phi(t) \ll 1$. Taking three initial terms of the expansion one gets

$$I(t) \simeq C\frac{I_0\tau_0^2}{S_{av}}\left(-\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t}\right) \tag{32}$$

where the constant C equals

$$C=\frac{1}{2L^3}\int_0^L\left(\int_0^x S(x')\,\mathrm{d}x'\right)^2\mathrm{d}x. \tag{33}$$

Since the function $\Phi(t)$ decreases in time, the formulae (31) and (32) describe the initial and final portions of the TSC curve, respectively. By equating these formulae one obtains the equation determining approximately the position of the TSC maximum, which corresponds to the effective carrier transit time τ_e throughout the sample:

$$\tau_0\Phi(\tau_e) \simeq C^{-1/2}. \tag{34}$$

The above equations have a similar form to those derived for surface carrier generation (Tomaszewicz *et al* 1990). The main difference is that the initial rise of the TSC, described by equation (31), does not depend on the spatial trap distribution. The constant C, defined by equation (33), shows a stronger dependence on the shape function $S(x)$ than in the case of surface initial generation. This suggests that in the present case the course of the TSC, except for its initial portion, is more strongly influenced by the spatial trap distribution than for surface generation.

As in the case of non-dispersive transport, analytical explicit solutions for case (ii) (condition (3*b*)) are available provided $S(x)$ is known. The formulae corresponding to exponential dependence of $S(x)$ are given in appendix 2.

3. Numerical results and discussion

In the present section we compare the analytical formulae with the results of the Monte Carlo simulation, which are considered as exact solutions of the transport equations. In order to proceed we must specify the manner of heating and the energetic and spatial distributions of traps. We shall assume the linear heating scheme $T(t) = T_0 + \beta t$, where T_0 is the initial temperature of the sample and β is the heating rate. As far as the energetic trap distribution is concerned, we shall use the discrete single level

$$N(\mathscr{E}) = N_0\delta(\mathscr{E} - \mathscr{E}_0) \tag{35}$$

for non-dispersive transport ($\mathscr{E}_0$ = trap depth), and the exponential distribution

$$N(\mathscr{E}) = (N_0/kT_c)\exp[-(\mathscr{E} - \mathscr{E}_t^0)/kT_c] \tag{36}$$

for dispersive transport. In equation (36) T_c is the characteristic temperature of the energetic trap distribution, $\mathscr{E}_t^0$ is the upper edge of the distribution and N_0 is chosen in such a way that the product $N_0S(x)$ is the total trap concentration in x. The model spatial trap distributions are chosen to be

$$S(x) = \exp(-x/D) \tag{37}$$

and

$$S(x) = \exp[-(L - x)/D]. \tag{38}$$

The Monte Carlo simulations were performed according to the previous algorithm with properly modified initial conditions (cf Tomaszewicz *et al* 1990).

Figures 1 and 2 present sample TSC curves for initial trapped carrier distribution proportional to the trap density (case (i)), for non-dispersive and dispersive transport, respectively. In figure 1 the Monte Carlo results are shown with full curves and the analytical solutions (18) with broken curves. Curve a corresponds to a spatially homogeneous trap distribution. Curves b and c have been calculated for trap density decreasing and increasing exponentially in x by a factor e^2. Curve d corresponds to the identical spatial trap distribution as for curve a, but under reversed direction of the external electric field. Curve e has been calculated for a spatially homogeneous trap distribution

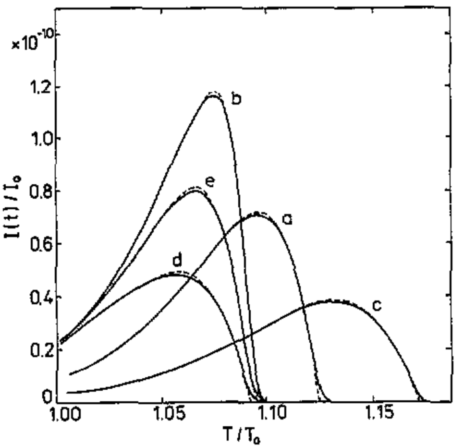

Figure 1. Non-dispersive TSC curves for a non-homogeneous spatial trap distribution, linear heating scheme, and initial condition (3*a*). Full curves correspond to Monte Carlo simulation; broken curves correspond to solution (18). Curves: a, trap distribution (37) with $L/D = 0$ (homogeneous trap distribution); b, $L/D = 2$; c, $L/D = -2$; d, trap distribution (38), $L/D = 2$; e, homogeneous trap distribution with the same mean trap density as for curves b and d. Here $\tau_0\beta/T_0 = 10^{-10}$, $\tau(x)S(x)\beta/T_0 = 2 \times 10^{-13}$, $\nu_0 T_0/\beta = 5 \times 10^{15}$, $\mathscr{E}_0/kT_0 = 30$.

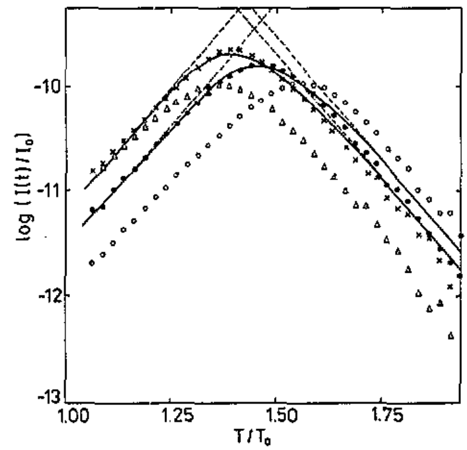

Figure 2. Dispersive TSC curves for a non-homogeneous spatial trap distribution, linear heating scheme, and initial condition (3*a*). Points are Monte Carlo simulation; full curves correspond to solution (30); broken lines correspond to approximate solutions (31) and (32)–(33). Curves: (●), trap distribution (37) with $L/D = 0$ (homogeneous trap distribution); (×), $L/D = 2$; (○), $L/D = -2$; (△), trap distribution (38), $L/D = 2$. Here $T_0/T_c = 0.33$, $\tau_0\beta/T_0 = 10^{-10}$, $\tau(x)S(x)\beta/T_0 = 1.187 \times 10^{-13}$, $\nu_0 T_0/\beta = 10^{15}$.

of density equal to the average value of that of curve b (or d). As is easily seen, the shape of the increasing portions of the currents depends distinctly on the spatial trap distribution. Curves b and d show a polarity dependence of TSC peaks. Moreover, in contrast to the case of surface generation (Tomaszewicz *et al* 1990), the position of the current maximum also depends on the spatial trap distribution (curves b, e and d). On the other hand, the shape of the current increase is somewhat less pronounced than in the case of surface initial generation of trapped carriers. Figure 2 corresponds to dispersive transport for initial condition (3*a*). Points represent the Monte Carlo results, full curves the analytical solution (30), and broken lines the approximate expressions for the initial increase and final decrease of TSC, according to (31) and (32)–(33), respectively (drawn only for two curves in order not to complicate the figure). As can be seen, the approximate expressions coincide with the exact solution except for in the vicinity of the TSC maximum, and the cross section of the lines given by (31) and (32)–(33) determines the position of the peak maximum relatively well.

The two remaining figures (figures 3 and 4) present sample TSC curves for uniform initial distribution of trapped carriers in a similar manner. The most important difference between the case of a uniform initial distribution of trapped carriers (case (ii) of the present work) and those of a non-uniform initial distribution of trapped carriers (e.g. case (i) of the present work or surface initial generation (Tomaszewicz *et al* (1990)) consists of the disappearance of the polarity effects. In figure 3, curve b represents two coinciding non-dispersive TSC obtained for exponentially decreasing and increasing densities of traps, with the same maximum value at $x = 0$ and $x = L$, respectively. In the

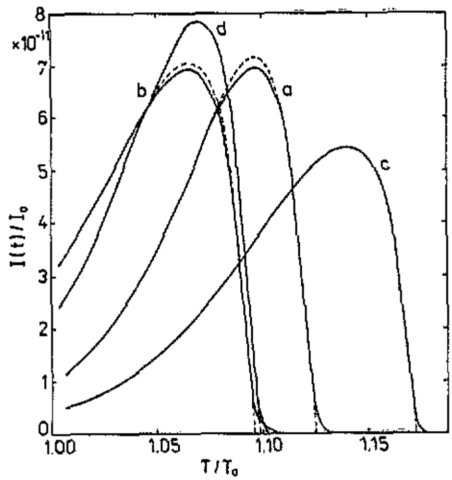

Figure 3. Non-dispersive TSC curves for a non-homogeneous spatial trap distribution, linear heating scheme, and initial condition (3*b*). Full curves correspond to Monte Carlo simulation; broken curves correspond to solution (A2.4) from appendix 2. Curves: a, trap distribution (37) with $L/D = 0$ (homogeneous trap distribution); b, two coinciding curves with parameters as for curves b and d in figure 1; c, $L/D = -2$; d, parameters as for curve e in figure 1. All other parameters also as in figure 1.

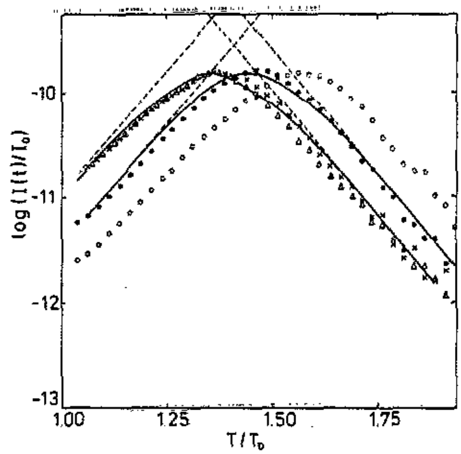

Figure 4. Dispersive TSC curves for a non-homogeneous spatial trap distribution, linear heating scheme, and initial condition (3*b*). Points are Monte Carlo simulation; full curves correspond to solution (A2.5); broken curves correspond to approximate solutions (A2.6)–(A2.7) and (A2.8)–(A2.9). Curves: (●), trap distribution (37) with $L/D = 0$ (homogeneous trap distribution); (×), $L/D = 2$; (○), $L/D = -2$; (△), trap distribution (38), $L/D = 2$. Here $T_0/T_c = 0.33$, $\tau_0\beta/T_0 = 10^{-10}$, $\tau(x)S(x)\beta/T_0 = 1.187 \times 10^{-13}$, $\nu_0 T_0/\beta = 10^{15}$.

case of dispersive transport the Monte Carlo results differ somewhat on polarity reversal, but theoretical curves remain unchanged (figure 4). The effect of polarity invariance of TSC is not an artifact related to the assumed (for our illustrative calculations) exponential form of $S(x)$, but is a general result valid for any $S(x)$ and x-independent density of initially trapped carriers (appendix 1). Curve d in figure 3 has been calculated for a spatially homogeneous trap distribution of density equal to the average value of that of curve b, and (in comparison with the latter) shows the effect of the trap distribution non-homogeneity. Similarly as in figures 1 and 2, the analytical formulae ((A2.4) for non-dispersive transport and (A2.5)–(A2.9) for dispersive transport) agree well with Monte Carlo results of figures 3 and 4.

4. Methods to determine the spatial trap distribution and concluding remarks

Methods of determining the spatial trap distribution on the basis of TSC measurements performed under the initial conditions considered here are similar to those described in our previous paper (Tomaszewicz *et al* 1990), and thus are not presented in detail. We shall give only a short scheme of the procedure. For an initial distribution of trapped carriers proportional to the local trap density and in the case of non-dispersive transport, the energetic trap distribution, and therefore the form of the function $\theta(t)$, can be determined from the dependence of the carrier transit time τ_e on L/E and/or β. If $n_t(x,0) \sim S(x)$ one can then determine the position of the carrier packet left tail $\bar{x}(t)$, and its time derivative $d\bar{x}(t)/dt$ from the shape of the TSC curve (cf equation (18)). Since $S[\bar{x}(t)] = \mu_0 E\theta(t)/[d\bar{x}(t)/dt]$ (cf equation (17)), we are able to get immediately the value of the function $S(x)/\mu_0$ at the point $x = \bar{x}(t)$. An analogous procedure may be applied in the case of dispersive transport. The energetic profile of traps and the shape of the function $\Phi(t)$ can be found from the dependence of the TSC maximum on L/E and β, or from the final decay of the TSC (cf equations (34) and (32)). The spatial trap distribution can be determined by the trial-and-error method, assuming an analytic form of the function $S(x)$. When $n_t(x, 0) \sim S(x)$, formula (30) for the TSC may be utilized for the TSC calculation.

For other initial distributions of the trapped carriers, for both non-dispersive and dispersive transport, one has to assume a concrete form of the function $S(x)$ with one or more parameters, and derive the formulae for TSC as in appendix 2. The values of the parameters may be obtained by fitting the calculated TSC curves to the experimental ones.

It must be stressed that the considered methods of determining the spatial trap distribution are expected to be reliable only if the initial carrier distribution in the sample is specified with sufficient accuracy. Moreover, the more homogeneous the distribution of initially trapped carriers, the less distinct is the influence of spatial non-homogeneity in the trap distribution on the TSC, the influence being most pronounced for surface initial generation (cf Tomaszewicz *et al* 1990). Thus, TSC measurements under initial condition (3*b*) (case (ii)) are particularly suitable to determine the energetic trap distribution, whereas measurements under initial condition (3*a*) (case (i)) or surface initial generation are particularly suitable for determination of the spatial trap distribution.

Acknowledgment

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Appendix 1

We prove here the polarity independence of TSC for a spatially non-homogeneous trap distribution and a uniform initial distribution of trapped carriers. Let us consider two shape functions $S_1(x)$ and $S_2(x)$, which obey $S_2(x) = S_1(L - x)$. Under our conditions we have on the basis of (14)

$$S_i(x) f_i[z_i(x)] = n_0 \tag{A1.1}$$

where

$$z_i = \frac{1}{\mu_0 E}\int_0^x S_i(x')\,dx' \qquad i = 1, 2. \tag{A1.2}$$

As is easily seen from (A1.1) and (A1.2)

$$f_2(z) = f_1[z(L) - z]. \tag{A1.3}$$

For non-dispersive transport the intensity of TSC (given by equations (6) and (15)) can be expressed as

$$I(t) = \frac{I_0 \theta(t)}{n_0^2 \tau_0} \int_{\zeta(t)}^{z(L)} f[z - \zeta(t)] f(z)\, dz \qquad \zeta(t) \leq z(L)$$
$$I(t) = 0 \qquad \zeta(t) > z(L). \tag{A1.4}$$

Thus we have

$$I_2(t) \sim \int_{\zeta(t)}^{z(L)} f_2[z'' - \zeta(t)] f_2(z'')\, dz'' = \int_{\zeta(t)}^{z(L)} f_1[z(L) - z'' + \zeta(t)] f_1[z(L) - z'']\, dz''$$
$$= \int_{\zeta(t)}^{z(L)} f_1[z' - \zeta(t)] f_1(z')\, dz' \sim I_1(t).$$

In the above sequence of equalities property (A1.3) has been used and the change of variables $z' = z(L) - z'' + \zeta(t)$ has been performed.

For dispersive transport, making use of equations (6), (23) and (25), and bearing in mind that $h(z) = f(z)$, we get

$$I(t) = \frac{I_0}{n_0^2 \tau_0} \frac{d}{dt} \int_0^{z(L)} f(z) \left(\int_0^z f(z - z') \exp[-z' \Phi(t)]\, dz' \right) dz$$
$$= \frac{I_0}{n_0^2 \tau_0} \frac{d}{dt} \int_0^{z(L)} \exp[-z' \Phi(t)] \left(\int_{z'}^{z(L)} f(z - z') f(z)\, dz \right) dz'. \tag{A1.5}$$

Because the inner integral in the last expression (in respect of z) has the same form as the integral (A1.4) determining $I(t)$ for non-dispersive transport, in the case of dispersive transport the TSC shape is also polarity-independent.

Appendix 2

We derive here the formulae determining TSC for a uniform initial distribution of trapped carriers (3b) and a spatial distribution of traps decreasing in x (equation (37)) in the cases of non-dispersive and dispersive transport. According to the results of appendix 1, the TSC corresponding to the spatial distribution increasing in x (equation (38)) are given by the same formulae.

As far as non-dispersive transport is concerned, equations (3b) and (14) give

$$S(x) f[z(x)] = n_0. \tag{A2.1}$$

If the spatial trap distribution is given by (37), the function $z(x)$, defined by (12), is

$$z(x) = \tau_D [1 - \exp(-x/D)] \tag{A2.2}$$

where $\tau_D = D/\mu_0 E$. From (37) and (A2.1) and (A2.2) one obtains

$$f(z) = n_0/(1 - z/\tau_D). \tag{A2.3}$$

According to (A1.4) the resulting TSC turns out to be

$$I(t) = [I_0 \tau_D \theta(t)/\zeta(t)] [\![1 + (D/L) \ln\{[1 - \zeta(t)/\tau_D][\exp(-L/D) + \zeta(t)/\tau_D]\}]\!] H(\tau_e - t). \tag{A2.4}$$

One can note that the initial current increase is given by $I(t) \sim \theta(t)$, and the effective transit time τ_e is determined by $\bar{x}(\tau_e) = L$, as in the case of $n_t(x, 0) \sim S(x)$ (cf section 2.2).

In the case of dispersive transport the resulting TSC, according to (A1.5) and (A2.3), is

$$I(t) = \frac{I_0 D\tau_D}{L\Phi(t)}\left(-\frac{d\Phi(t)}{dt}\right)\int_0^{z(L)} \exp[-z'\Phi(t)] \times [\![1 + (D/L)\ln\{(1 - z'/\tau_D)[\exp(-L/D) + z'/\tau_D]\}]\!]\, dz'. \quad \text{(A2.5)}$$

In the limiting case of $z(L)\Phi(t) \gg 1$

$$I(t) \simeq C_0 I_0 \frac{d}{dt}\left(\frac{1}{\Phi(t)}\right) \quad \text{(A2.6)}$$

where

$$C_0 = (D/L)[\exp(L/D) - 1]. \quad \text{(A2.7)}$$

On the other hand, if $z(L)\Phi(t) \ll 1$, then

$$I(t) \simeq C I_0 \tau_0^2[-d\Phi(t)/dt] \quad \text{(A2.8)}$$

where

$$C = (D/L)^2[1 - 2D/L + (1 + 2D/L)\exp(-L/D)]. \quad \text{(A2.9)}$$

Equations (A2.6) and (A2.8) describe the initial rise and the final decay of the TSC, respectively. They are of the same form as equations (31) and (32), except for the values of the multiplicative constants.

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